

Complex Impedance Meter

Notes by Guy Fletcher VK2KU

The idea for this instrument was first described by Carl G. Lodström in RF Design, December 1986, pages 50-53, and realized in working form by Gordon VK2ZAB. I have spent some time analyzing the operation of this instrument, and Gordon has asked me to place these notes along with his article on the NSW VHF Dx Group web page.

The instrument uses a 50Ω coaxial transmission line with four sampling points at intervals of $l/8$ along the line. Diodes are connected to each point to detect the voltage, with capacitors to smooth the detected signal. I have labelled the sampling points A, B, C, D working *backwards* from the output connector (the reference plane); A is located $l/8$ from the output socket.

The complex reflection coefficient r at a complex load $Z = R + jX$ is given by $Z = (Z - Z_0) / (Z + Z_0)$ with $Z_0 = 50\Omega$.

If the forward wave reaching the load has amplitude V_0 , then:

the peak-rectified voltage at A is $V_A = V_0 |1 - jr|$

the peak-rectified voltage at B is $V_B = V_0 |1 - r|$

the peak-rectified voltage at C is $V_C = V_0 |1 + jr|$

the peak-rectified voltage at D is $V_D = V_0 |1 + r|$.

The diodes are oriented so as to subtract pairs of voltages.

The voltage applied to the “resistance” meter is $(V_D - V_B)$, and

the voltage applied to the “reactance” meter is $(V_A - V_C)$.

The resulting expressions for the two voltages can easily be worked out, but they are somewhat messy! Each one depends on both R and X. In other words you cannot expect one meter to read only the R component of the load, and the other meter only the X component. However the behaviour of the two meters as the load is varied is extremely helpful in adjusting a complex load such as an antenna.

1. Pure Resistive Load

With a resistive load of 50Ω there is no reflected wave, so the voltage is the same at all points along the line. Subtraction of the pairs of voltages gives zero at each meter (the centre point of each scale).

With a short circuit at the load, the forward wave is completely reflected in antiphase, giving a node at the load. The resulting standing wave has voltage $2V_0$ at an antinode (B), and zero at a node (D). The voltage applied to the R meter is $-2V_0$, which is set to the left hand end of the meter. Points A and C each fall midway between node and antinode, and their voltages are therefore equal, so the voltage applied to the X meter is zero (the centre point of the scale).

With an open circuit at the load, the forward wave is completely reflected in phase, giving an antinode at the load. The resulting standing wave has voltage $2V_0$ at an antinode (D), and zero at a node (B). The voltage applied to the R meter is $+2V_0$,

which now corresponds to the right hand end of the meter. Points A and C again fall midway between node and antinode, so the voltage applied to the X meter is zero (the centre point of the scale).

For other values of resistive load, partial reflection occurs at the load, in antiphase for $R < 50\Omega$ (node at the load and at D), and in phase for $R > 50\Omega$ (antinode at the load and at D). The voltage at the antinode is less than $2V_0$, and that at the node is greater than zero. So the voltage applied to the R meter is less than $2V_0$ in magnitude; it is equal to $2V_0 \left(\frac{R - 50}{R + 50} \right)$. This expression indicates how to calibrate the R meter scale.

The voltage applied to the X meter is zero for all values of purely resistive load.

2. Pure Reactive Load

For all values of purely reactive load the forward wave is completely reflected at the load, giving a standing wave with $2V_0$ at an antinode and zero at a node. However the position of the nodes and antinodes depends on the magnitude and sign of the reactance.

For $X = +50\Omega$ (inductive) at the load the reflected wave is shifted in phase by $+90$ degrees. As a result the antinode falls at A and the node at C, giving a voltage applied to the X meter equal to $2V_0$ (maximum positive deflection). For $X = -50\Omega$ (capacitive) the phase shift is -90 degrees, putting the node at A and the antinode at C, and giving a voltage to the X meter of $-2V_0$ (maximum negative deflection). In both cases points B and D lie midway between node and antinode, making their voltages equal, so the R meter reads zero.

As the magnitude of the reactance is reduced below 50Ω , the standing wave pattern shifts sideways, so that A and C no longer correspond to the node or antinode, and the difference between them falls - the reading of the X meter is reduced (towards the centre zero). However points B and D no longer lie midway between node and antinode, and these two voltages are no longer equal in magnitude; so there is a reading on the R meter even though the load has *no* resistive component. In the extreme case of $X = 0$ the R meter will read maximum negative, as you might expect for a short circuit load, and the X meter will read zero (centre scale).

A similar thing happens as the magnitude of the reactive load is increased above 50Ω . The standing wave pattern shifts sideways in the opposite direction so that A and C no longer correspond to the node or antinode. The reading of the X meter is *reduced*, and again the R meter starts to read but in a positive direction (right of centre). As X becomes infinitely large (open circuit), the X reading falls to zero (centre scale) and the R meter reads maximum positive.

Two things need to be emphasized. For a pure reactive load not equal to 50Ω in magnitude, the R meter will deflect when there is no resistive component in the load. And for values of reactance greater than 50Ω in magnitude, the scale of the X meter folds back on itself (from left or right) until the reading for infinite X is the same as for zero X (centre scale). The maximum deflection of the X meter occurs for $X = \pm 50\Omega$.

For positive deflection and $X \leq 50\Omega$ the voltage applied to the X meter is

$$2V_0 \left(\frac{X\sqrt{2}}{\sqrt{X^2 + 50^2}} \right), \text{ which peaks at } 2V_0 \text{ when } X = 50\Omega.$$

For positive deflection and $X \geq 50\Omega$ the voltage applied to the X meter is

$$2V_0 \left(\frac{50\sqrt{2}}{\sqrt{X^2 + 50^2}} \right).$$

These two expressions provide the basis for calibrating the X meter scale.

3. Resistive and Reactive Load

For a general load having both resistance and reactance the nodes and antinodes will not correspond with any of the four sampling points, and both meters will deflect from their central position. However the scale readings will *not* correspond properly with the load value because *both* readings depend on *both* R and X. For example with a load of $Z = (60 + 10j)\Omega$, the R meter will read 61Ω and the X meter will read 2.9Ω .

The R meter reading is only slightly affected by the relatively small reactive component, whereas the X meter reading is greatly reduced by the large resistive component. However the R meter reading would be much more in error if the reactive component was as high as say 50Ω , as might well be the case for initial measurement of an antenna; the R meter would then read 80Ω .

For practical antenna adjustment these are not serious problems. If the antenna is first adjusted to reduce the X meter reading to zero, then the reactive component will indeed be zero and the R meter reading will then be accurate.

Both meters correctly record departures from a perfect 50Ω impedance with respect to direction and phase, even if neither reading is accurate. Of course any actual adjustment to the antenna will almost certainly affect the values of both resistive and reactive components anyway - that's how antennas behave!

Appendix

Example 1: $Z = R + jX$ with $R = 50\Omega$

For $X = 0\Omega$,	R meter reads 50Ω ,	X meter reads 0Ω .
For $X = \pm 10\Omega$,	R meter reads 51Ω ,	X meter reads $\pm 3.5\Omega$.
For $X = \pm 50\Omega$,	R meter reads 73Ω ,	X meter reads $\pm 14.4\Omega$.

Example 2: $Z = R + jX$ with $R = 60\Omega$

For $X = 0\Omega$,	R meter reads 60Ω ,	X meter reads 0Ω .
For $X = \pm 10\Omega$,	R meter reads 61Ω ,	X meter reads $\pm 2.9\Omega$.
For $X = \pm 50\Omega$,	R meter reads 80Ω ,	X meter reads $\pm 12.1\Omega$.